



Conversions and Constants

1 kPa = 0.1450 psi	1 in = 2.54 cm	1 acre = 43560 ft ²	1 m ³ = 6.2898 bbl	R = 459.67 + °F	1 lb _m = 453.59 g
1 MPa = 10 bar	1 ft = 0.3048 m	1 m ² = 10.764 ft ²	1 bbl = 5.6146 ft ³	K = 273.15 + °C	1 cp = 1 mPa-s
1 atm = 14.696 psi	1 mile = 5,280 ft	1 ft ³ = 7.4805 gal	1 bbl = 42 US gal	°F = 1.8 °C + 32	
1 atm = 1.013 bar	1 mile = 1,609 m				
1 bar = 1 x 10 ⁶ dynes/cm ²	1 Darcy = 9.8692 x 10 ⁻⁹ cm ²	Gas Constant = 10.732 psia · ft ³ /lbmol · R		Water density at SC = 62.37 lb _m /ft ³	
1 Newton = 1 x 10 ⁷ dynes	1 mD/CP = 6.33 x 10 ⁻⁹ ft ² /psi-day	Water density at SC = 62.37 lb _m /ft ³		Molar Mass of Air = 28.966 g/mol	
1 dyne = 2.248 x 10 ⁻⁶ lb _f	1 lb _m /gal = 0.052 psi/ft	V _M = 379.3 scf/lbmol @ 14.696 psia			
Standard Temperature = 60°F	Standard Pressure = 14.696 psia				

Single Phase Flow

Continuity Equation

Mass balance leads to Continuity Equation
(Mass of fluid in) - (Mass of fluid out) = (Fluid accumulation)

$$-\nabla \cdot (\rho u) = \frac{\partial(\rho \phi)}{\partial t}$$

Darcy's Law

$\phi = P + \rho gh$ $u = \frac{q}{A}$ $u = -\frac{k}{\mu} \nabla \phi$

Fluid flows from higher potential to lower potential (not pressure)

ϕ = flow potential q = flow rate [=] L³/t
 u = fluid velocity [=] L/t μ = viscosity
 k = permeability [=] L²
 A = cross sectional area perpendicular to flow

Water flows towards oil zone

1-Dimensional Flow

Incompressible Fluid - Flowing at Steady State

Horizontal Flow: $u = -\frac{k}{\mu} \frac{dP}{dx}$

Non-Horizontal Flow: $\phi_2 = P_2$

Radial Flow: $u = -\frac{k}{\mu} \frac{dP}{dr}$ $A = 2\pi rh$

Slightly Compressible Fluid

Horizontal flow at steady state

Compressibility $c = -\frac{1}{V} \frac{\partial V}{\partial P} = \frac{1}{\rho} \frac{\partial \rho}{\partial P}$

$\rho = \rho_0 \exp(c \Delta P) \approx \rho_0 + \rho_0 c P$

Rewrite Darcy's Law

$q_0 = -\frac{kA}{\mu} \frac{dP}{dx}$

Lab Measurement of Porosity

Boyle's Law: $P_1 V_1 = P_2 V_2$

Most common method for measuring V_p

$P_1(V - V_c) = P_2(V - V_c + \Delta V)$

Deviations from Darcy's Law

High Flow Rates

Re > 1 $Re = \frac{\rho u L}{\mu}$ $L = \sqrt{k/\phi}$

Forchheimer Equation

$\frac{dP}{dx} = \frac{\mu}{k} u + \beta u^2$

β = inertial flow coefficient
 β depends on pore structure

Non-Newtonian Fluid

viscosity not constant $\tau \neq \mu \dot{\gamma}$

Power Law Fluid

apparent viscosity $\mu_{app} = a \dot{\gamma}^{n-1}$

$\dot{\gamma}$ = shear rate a = consistency index
 n = flow behavior index - typically 0.4 - 0.7

$q = -\frac{kA}{\mu_{app}} \left(\frac{dP}{dx}\right)$

Klinkenberg Effect

gas flow at low pressure

$k_g = k_L \left(1 + \frac{b}{P_m}\right)$

k_g = gas k_L = liquid k

b factor important when $k < 10$ md

P_m = mean flowing pressure

Average Permeability

Parallel Layers

Average Permeability $\bar{k} = \frac{\sum h_i k_i}{\sum h_i}$

Layers in Series

Linear Flow: $\bar{k} = \frac{\sum L_i}{\sum L_i/k_i}$

If horizontal and vertical variations, can use Geometric Average $\bar{k} = (k_1 k_2 \dots k_n)^{1/n}$

Multiphase Flow - Relative Permeability

Darcy's Law

$u_j = -\frac{k k_{rj}}{\mu_j} \left(\frac{dP}{dx}\right)$ j = phase

$k_{rj} = \frac{k_j}{k}$ = relative permeability

Wettability

the strongest influence on k_{rj}

Water-Wet, Oil-Wet, Non-wetting phase occupies larger pores so it flows easier

3-Phase Permeability

k_{rW} is only a function of S_w

$k_{rW}(S_w, S_{NW})$ $k_{rNW}(S_w, S_{NW})$

Relative Permeability Curves

Water-Wet Rock: $S_w > 0.5$ when curves intersect and $k_{rW}^o < k_{rOW}^o$ (inverse for Oil-Wet)

Influences on Relative Permeability

Effect of Pore Structure: High Perm Sandstone, Low Perm Sandstone

Effect of Interfacial Tension: Minimal σ , reducing S_{wr} and S_{nrw}

Lab Measurement of Relative Permeability

Steady State (more reliable) and Unsteady State (more common) methods

Steady State flows two fluids simultaneously

Unsteady State displaces a fluid by injecting another

Sedimentary Rock Properties

Class	Formation	Name	Mineralogy	ρ (g/cc)
Clastic	Rock fragments compacted and cemented together	Sandstone	Quartz	2.65
		Shale	Clay minerals	2.2-2.7
		Limestone	Calcite	2.71
Carbonate	Dissolution and precipitation of calcite	Dolomite	Dolomite	2.87
		Gypsum	Gypsum	2.32
		Anhydrite	Anhydrite	2.98
Evaporite	Evaporation of water	Salt	--	1.87-2.03
		Coal	--	0.64-0.93
		Peat	--	--
Organic	Accumulation of organic debris	Diatomite	--	0.224

Clay Mineralogy

Clay Group	Structure	Swelling
Kaolinite	1:1	No
Smectite	2:1	Yes
Illite	2:1	No
Chlorite	2:2	No

2:1 structure lacks hydrogen bonding

Porosity

Porosity Definition

ratio of pore volume to bulk volume

$\phi = \frac{V_p}{V_b} = 1 - \frac{V_s}{V_b}$ V_s = solid volume

Effective Porosity

interconnected porosity

residual $\phi = \phi_R = \phi_T - \phi_E$

ϕ_T = total or absolute ϕ_E = effective

Packing Structures

Cubic Packing: $V_b = (2r)^3 = 8r^3$
 $V_s = (4/3)\pi r^3$
 $\phi = 1 - (4/3)\pi r^3 / 8r^3 = 0.476$

Hexagonal Close Pack: $V_b = 6(V_{sphere}) = 8\pi r^3$
 $V_s = A_{base} h_{cell}$
 $V_b = (6\sqrt{3}\sqrt{3})(4r\sqrt{2/3})$
 $\phi = 1 - \frac{8\pi r^3}{24r^3\sqrt{2}} = 0.26$

The Bundle of Tubes Model

Hagen-Poiseuille

Flow in a single tube

$q_i = \frac{\pi r^4 \Delta P}{8\mu l}$

$A = \pi r^2$

$u = \frac{r^2 \Delta P}{8\mu l}$

For 'n' parallel tubes, the total flow rate

$q_T = \left(\frac{\pi r^4 \Delta P}{8\mu l}\right) n$

Porosity and Permeability

Pore Volume = $\phi = \frac{n(\pi r^2 l)}{AL}$

Total Volume = $\frac{\phi AL}{n}$

$n = \frac{\phi AL}{\pi r^2 l}$

$q_T = \left(\frac{\pi r^4 \Delta P}{8\mu l}\right) \left(\frac{\phi AL}{\pi r^2 l}\right) = \frac{k \Delta P}{\mu L}$

Empirical Approximation $\tau = \frac{25}{12}$

$k = \frac{d_p^2 \phi^3}{72\tau(1-\phi)^2}$

Blake-Kozeny Equation

Bundle of Tubes: $\frac{SA}{V} = \frac{2\pi r l}{\pi r^2 l} = \frac{2}{r}$

Packed bed of spherical particles

$\frac{SA}{V} = \frac{\pi d_p^2}{6\pi d_p^3} \left(\frac{\phi}{1-\phi}\right) = \frac{6}{d_p} \left(\frac{1-\phi}{\phi}\right)$

Equate the two models above

$k = \frac{d_p^2}{8\tau} \left(\frac{\phi}{1-\phi}\right)^2$ $\leftarrow k = \frac{d_p}{3} \left(\frac{\phi}{1-\phi}\right)$

$k \approx \frac{d_p^2 \phi^3}{150(1-\phi)^2}$

Interfacial Tension

Force Balance

$\sigma_{os} = \sigma_{ws} + \sigma_{ow} \cos \theta_c$ [=] Force/Length

σ_{ij} = interfacial tension between phases i and j

Contact Angle

$\cos \theta_c = \frac{\sigma_{os} - \sigma_{ws}}{\sigma_{ow}}$

θ_c = contact angle

Oil-Wet Surface: $\theta_c > 90^\circ$
 $\sigma_{os} < \sigma_{ws}$

Intermediate-Wet: $\theta_c \approx 90^\circ$
 $\sigma_{os} \approx \sigma_{ws}$

Water-Wet Surface: $\theta_c < 90^\circ$
 $\sigma_{os} > \sigma_{ws}$

Capillary Pressure

Laplace Equation

$P_c = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

R = radius of curvature

Spherical Interface: $R_1 = R_2 = \frac{r}{\cos \theta}$

$P_c = \frac{2\sigma}{R} = \frac{2\sigma \cos \theta}{r}$

$P_{res} = \frac{(\sigma \cos \theta)_{res}}{(\sigma \cos \theta)_{lab}} P_{lab}$

Capillary Rise

$P_c = 2\sigma/R$ $r = R \cos \theta$ $P_c = \frac{2\sigma \cos \theta}{r}$

$P_c = (2\sigma \cos \theta)/r = (\Delta \rho_f l_{fluid}) g h$

$h = (2\sigma \cos \theta) / (r g \Delta \rho_f l_{fluid})$

Capillary Pressure Curves for Rocks

$P_c(S_w) = P_{rNW} - P_w$ nw = non-wetting phase

Depth vs Water Saturation Curve

$P_c(S_w) = (\rho_w - \rho_o) g h$

Oil-water contact is depth at which S_w first deviates from 1

Normalizing Capillary Pressure Curves

$J(S_w) = \frac{P_c}{\sigma} \sqrt{\frac{k}{\phi}}$ $\frac{P_{c1}}{P_{c2}} = \frac{\sqrt{k_2/\phi_2}}{\sqrt{k_1/\phi_1}}$

Elastic Properties of Rocks

Basic Definitions

Hooke's Law $\sigma = -E \epsilon$

$\sigma = \frac{F}{A}$ $\epsilon = \frac{\Delta L}{L}$

$E = \text{Young's Modulus} [=] \text{psi}$

$\nu = \text{Poisson's Ratio}$

$\nu = \text{transverse expansion} / \text{axial compression}$

Linear Elasticity

Bulk modulus $K = \frac{E}{3(1-2\nu)}$

$\epsilon_x = \left(\frac{1+\nu}{E}\right) \sigma_x - \frac{\nu}{E} (\sigma_x + \sigma_y + \sigma_z)$

$i = x, y, \text{ or } z$

Shear modulus $G = \frac{E}{2(1+\nu)}$

Subsurface Stresses

$\sigma_v, \sigma_{Hmax}, \sigma_{Hmin}$ are the principal stresses

σ_v = vertical stress $\sigma_v = \rho_b g h$ ρ_b = bulk density

Neglecting tectonic stresses

$\sigma_x = \sigma_y = \sigma_H = \left(\frac{\nu}{1-\nu}\right) \sigma_v + P_p \left(\frac{1-\nu}{1-\nu}\right)$

σ_H = horizontal stress P_p = pore pressure

Tectonic stresses create differences between σ_{Hmin} & σ_{Hmax}

Fracture orientation controlled by direction of σ_{Hmin}